

Using Public Records to Support the Productive Use of Student Mathematical Thinking

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A public record is ...



- A visual representation of mathematical thinking
- Publicly accessible to all participants within the classroom

Disagree
b/c if x is a
negative number

$$x = -1$$

$$1x = 1 \cdot -1 = -1$$

$$2x = 2 \cdot -1 = -2$$

$$-2 < -1$$

if $x = 0$

$$2x = 2 \cdot 0 = 0$$

$$1x = 1 \cdot 0 = 0$$

$$0 = 0$$

if x is a positive
number

$$x = 27$$

$$2x = 2 \cdot 27 = 54$$

$$1x = 1 \cdot 27 = 27$$

$$54 > 27$$

Education is the most powerful weapon which you can use to change the world!

FOCUS!!!

How might you argue this holds up mathematically?

HOMEWORK

- Math p. 257
- Sci → bring plant
- NO CLASS ON FRIDAY

Quiz

① $\frac{0}{4} + \frac{0}{6} + \frac{0}{3} = ?$

② $7\frac{4}{8} + \frac{6}{10} = ?$

③ $\frac{13}{4} \div \frac{1}{2} = ?$

④ Expand $(2x-5)(x+3)$

⑤ Solve;

$$6(x-4) + 3(x+7) = 3$$

$$6x - 24 + 3x + 21 = 3$$

$$9x - 3 = 3$$

$$9x = 6$$

$$x = \frac{2}{3}$$

Question: Which is larger, x or $x+x$?

what if we tried negative number, like $x = -5$?

$x+x$ is smaller

$x+x$ is ~~smaller~~ bigger??

$x+x$ is like $1x$ plus $1x$ which is bigger because

$2x$, so, it's greater than x , which is $1x$.

Then we would get $-5 + -5 = -10$

Do you agree or disagree with this??

Related Research



- Teachers' use of publicly accessible media (e.g., blackboards, whiteboards, etc.) is a widely accepted practice (Villareal & Borba, 2010).
 - Support student mathematical activity (Koehne et al., 2020; TIMSS 1995 Video Study)
 - Maintain continuity during collaborative inquiry (Staples, 2007)
 - Engaging with each others' ideas (Webb et al., 2014)
- Teaching practices that use student thinking can be challenging to enact (e.g., Simpson & Haltiwanger, 2017; Peterson & Leatham, 2009).

Research Question



- How can teachers use public records to support productive use of student mathematical thinking?

Research Question



- How can teachers use public records to support **productive use of student mathematical thinking**?

Research Question



- How can teachers use public records to support **productive use of student mathematical thinking**?
- How can teachers use public records to support **the teaching practice of *building on MOSTs***?

A MOST is a **M**athematical **O**pportunity in **S**tudent **T**hinking



Building on a MOST

Building on a MOST is engaging the class in making sense of the MOST to better understand the mathematics of the MOST.

Research Question



- How can teachers use public records to support **productive use of student mathematical thinking**?
- How can teachers use public records to support **the teaching practice of *building on MOSTs***?
- How can teachers use public records to support **the elements of the teaching practice of building on MOSTs**?

Framing the Use of Public Records



- Cognitive Load Theory (Swellers 1988; Swellers et al., 2011)
 - Long Term Memory & Working Memory
 - Extraneous Load & Intrinsic Load
- Ways instruction can reduce cognitive load
 - Integrating sources of information
 - Pairing visual display with speech

Methods



- Study is part of a larger project conceptualizing the building teaching practice
- Participants are grades 6-12 teacher researchers ($n=14$) who were enacting our conceptualized building practice and helping us to refine it
- Video-recordings of classroom enactments of four mini-tasks ($n=27$)
 - Percent Discount task - “The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?”
- We examined collections of *manipulating* and *referencing* across the enactments for of the four elements of building

Using a Public Record to Support the Establish Element



Manipulating

- Hone to clarify a MOST
- Placement to setup the public record for additional ideas
- Name or label the MOST helps make it an object

Referencing

- Minimal

The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

Claim:

The price will increase and decrease by the same amount.

I said yes, because, you know, the price of a necklace that you buy at a store that has that 50% deal in the problem will get increased and decreased by the same amount.

Using a Public Record to Support the Grapple Toss Element

Manipulating

- Minimize manipulating during Grapple Toss
- Emphasize/Highlight the MOST

Referencing

- Orient students to the details of the MOST

The price of a necklace was first increased 50% and then decreased 50%.
Is the final price the same as the original price?

Claim:

The price will increase
and decrease by the
same amount.

How might you
argue whether
this claim
holds up
mathematically?

VS.

How might you argue
whether **the price will
increase and decrease
by the same amount**
holds up mathematically?

Using a Public Record to Support the Conduct Element



Manipulating

- Similar actions as Establish
- Purposefully Organize

The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

Claim:

The price will increase
and decrease by the
same amount

$$\begin{aligned} \text{necklace} &= \$100 \\ \text{inc by } 50\% &= 50\% \text{ of } 100 = 50 \\ 100 + 50 &= \$150 \\ 50\% \text{ of } 150 &= \$75 \end{aligned}$$

$$\begin{aligned} \text{necklace} &= \$20 + 10 = 30 \\ \$30 - 10 &= 20 \end{aligned}$$

$$\begin{aligned} 50\% \text{ of } \$20 &= 10 \\ \$20 + 10 &= 30 \end{aligned}$$

$$\begin{aligned} 50\% \text{ of } 30 &= \$15 \\ 30 - 15 &= \$15 \end{aligned}$$

Using a Public Record to Support the Conduct Element



Manipulating

- Similar actions as Establish
- Purposefully Organize

The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

Claim:

The price will increase and decrease by the

Same amount
necklace = \$100
inc by 50% = 50% of 100 = 50
100 + 50 = \$150
50% of 150 = \$75

necklace = \$20 + 10 = 30
\$30 - 10 = 20

50% of \$20 = 10
\$20 + 10 = 30

50% of 30 = \$15
30 - 15 = \$15

Using a Public Record to Support the Conduct Element



Manipulating

- Similar actions as Establish
- Purposefully Organize
 - Distinguish among ideas

The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

Claim:

The price will increase
and decrease by the
same amount

necklace = \$100
inc. by 50% = 50% of 100 = 50
100 + 50 = \$150
50% of 150 = \$75

necklace = \$20 + 10 = 30
\$30 - 10 = 20

50% of \$20 = 10
\$20 + 10 = \$30

50% of 30 = 15
30 - 15 = \$15

1
2

Using a Public Record to Support the Conduct Element



Manipulating

- Similar actions as Establish
- Purposefully Organize
 - Distinguish among ideas
 - **Consider the placement of ideas**

The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

Yes

No

Claim:

The price will increase and decrease by the same amount.

necklace = \$100
inc. by 50% = 50% of 100 = 50
100 + 50 = \$150
50% of 150 = \$75

necklace = \$20 + 10 = 30
\$30 - 10 = 20

50% of \$20 = 10
\$20 + 10 = \$30
50% of 30 = 15
30 - 15 = \$15

Using a Public Record to Support the Conduct Element



Manipulating

- Similar actions as Establish
- Purposefully Organize
 - Distinguish among ideas
 - Consider the placement of ideas
 - **Seek parallelism of ideas**

The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

Yes

Claim:

The price will increase and decrease by the same amount.

necklace = \$20
50% of \$20 is \$10
inc. by 50%: \$20 + \$10 = \$30
dec. by 50%: \$30 - \$10 = \$20

No

necklace = \$100
50% of \$100 is \$50
inc. by 50%: \$100 + \$50 = \$150
50% of \$150 is \$75
dec. by 50%: \$150 - \$75 = \$75

necklace = \$20
50% of \$20 is \$10
inc. by 50%: \$20 + \$10 = \$30
50% of \$30 is \$15
dec. by 50%: \$30 - \$15 = \$15

Using a Public Record to Support the Conduct Element



Manipulating

- Similar actions as Establish
- Purposefully Organize
 - Distinguish among ideas
 - Consider the placement of ideas
 - Seek parallelism of ideas

Referencing

- Orient students to ideas that are being connected and synthesized
- Recenter student(s) on the MOST

The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

Yes

Claim:

The price will increase and decrease by the same amount.

necklace = \$20
50% of \$20 is \$10
inc. by 50%: \$20 + \$10 = \$30
dec. by 50%: \$30 - \$10 = \$20

No

necklace = \$100
50% of \$100 is \$50
inc. by 50%: \$100 + \$50 = \$150
50% of \$150 is \$75
dec. by 50%: \$150 - \$75 = \$75

necklace = \$20
50% of \$20 is \$10
inc. by 50%: \$20 + \$10 = \$30
50% of \$30 is \$15
dec. by 50%: \$30 - \$15 = \$15

Using a Public Record to Support the Make Explicit Element



Manipulating

- Edit the initial idea to help resolve the MOST

Referencing

- Emphasize the MOST as the object of a check-in question
- Orient students to the details of ideas and connections to help resolve the MOST

The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?

Yes

Claim:

The price will increase and decrease by the same amount.

necklace = \$20
50% of \$20 is \$10
inc. by 50%: \$20 + \$10 = \$30
dec. by 50%: \$30 - \$10 = \$20

No

necklace = \$100
50% of \$100 is \$50
inc. by 50%: \$100 + \$50 = \$150
50% of \$150 is \$75
dec. by 50%: \$150 - \$75 = \$75

necklace = \$20
50% of \$20 is \$10
inc. by 50%: \$20 + \$10 = \$30
50% of \$30 is \$15
dec. by 50%: \$30 - \$15 = \$15

Conclusion



- Purposefully organizing (*distinguishing, placement, parallelism*) the public record can help establish and sustain a common ground (Staples, 2007)
- Teacher use of public records can reduce extraneous cognitive load (Sweller et al., 2011)
- Referencing the public record helps orient students to details of each other's thinking, which can be key for learning (Webb et al., 2014)



Establish

Grapple
Toss

Conduct

Make
Explicit

Building on a MOST

Discussion Questions



- Questions for us?
- How might these uses of public records to support the building practice support other teaching practices?
- How might uses of public records differ depending on the teaching practice in which a teacher is engaged?

Thank you!



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